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An Optimization Model through Designing a Queuing Network for a Restaurant: A Case Study

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Abstract

Queuing theory has been gaining its attention day by day in the service industry. In this paper, a queuing network for a restaurant in Bangladesh has been designed to minimize the overall expected cost per unit of time through finding the optimal number of servers required to serve customers within satisfactory waiting time. This designed queuing network is comprised of three service stations, each of which corresponds to a certain queuing model. First queue is for giving order and paying bill; second one is for delivering the orders to the corresponding ovens for food preparation; third one is the queue of prepared foods waiting for a waiter available to be served to the corresponding customers. All the three queues are modeled as M/M/s. As the restaurant is designed to serve different types of foods, which are categorized based on their cooking processes - where each category itself contains many food items, multiple M/M/s models with different parameters have been used for the parallel queues in food preparation. As decision variables of the formulated optimization problem are integers and constrains are satisfying the waiting time to the desired level in each corresponding queuing model, genetic algorithm is used to solve the formulated nonlinear integer programming problem. The solution of the formulated model gives the optimum number of cash counters with the operating personnel, number of different types of ovens, and number of waiters to be appointed during the pick hours; all of which can be used to make pivotal business decisions.

Keywords: Queuing system, Queuing network, M/M/s, Optimization Model

Nomenclature

S_1	Number of servers at service facility 1
S_{2i}	Number of servers at service facility 2 for food class i
S_3	Number of servers at service facility 3
L_1	Length of queue at service facility 1
L_{2i}	Length of queue at service facility 2 for food class i
L_3	Length of queue at service facility 3
E(TC)	Expected total cost per hour
CS_n	Marginal cost at service facility n per hour
Cq_n	Cost incurred for queue at service facility n per hour
C_1	Marginal cost at service facility 1 per server per hour
C_{2i}	Marginal cost at service facility 2 for food class i per server per hour
C_3	Marginal cost at service facility 3 per server per hour
C_w	Equivalent loss of business per amount of waiting
F_{2i}	Percentage of foods under each of the food class i arrived per hour at service facility 2
W_1	Waiting time at service facility 1
W_{2i}	Waiting time at service facility 2 for food item i
W_3	Waiting time at service facility 3

q_1	Utilization factor at the service facility 1
q_{2i}	Utilization factor at the service facility 2 for food class i
q_3	Utilization factor at the service facility 3
λ_1	Mean arrival rate at service facility 1
λ_{2i}	Mean arrival rate at service facility 2 for food class i
λ_3	Mean arrival rate at service facility 3
μ_1	Mean service rate at service facility 1
μ_{2i}	Mean service rate at service facility 2 for food class i
μ_3	Mean service rate at service facility 3
ub_1	Upper bound of the number of servers at service facility 1
ub_{2i}	Upper bound of the number of servers at service facility 2 for food class i
ub_3	Upper bound of the number of servers at service facility 2
P_n	Probability of exactly n customers in queuing system

1. Introduction

In the recent days, restaurant business is flourishing all over the globe, mostly because of the increasing levels of urbanization, digitization and globalization. Although it is a costlier and a less salubrious way for the customers to fulfill their need for food, the restaurant industry is growing incessantly [1]. For a developing country like Bangladesh, the restaurant business has a very potential market as people are now more prone and coerced to eat out because of changes in culture and lifestyles [2]. Currently, this industry is facing intensified competition as more entrepreneurs want to participate in this profitable business. However, the profitability and success of a particular restaurant entirely relies on the satisfaction of the customers [3]. In these circumstances, apart from the price and quality of the foods, customer service is a crucial factor which generates customer satisfaction and loyalty ([4], [5]). The competing restaurants in Bangladesh have more or less achieved similar positions in terms of price charged and quality provided. Consequently, customer service plays its role as the cardinal factor behind the success of a restaurant in this industry. The most crucial parameter, in this case, upon which the service level and therefore, customer satisfaction depends is the waiting time of the customers in the queue [6]. Furthermore, waiting time of the customers at the table to receive their ordered food is also a significant factor. Among the competing restaurants in the country, the restaurants, where customers will have to spend least time in the queue and in the dine-in tables, will certainly be ahead of others. Such restaurants will be more fortuitous when it comes to keep loyal customers as well as attract new ones [7]. Increasing the number of servers at the counter, burners and ovens at the kitchen and waiters to serve the waiting customers will aid in reducing the waiting time of the customers. However, attempts to introduce more servers, burners and waiters will increase the operating cost which engenders a trade-off between the customer satisfaction and cost.

Considering the backdrop, in this research, queuing theory is utilized to generate a queuing network with the objective of minimizing the total cost per unit of time keeping an optimal number of servers, ovens and waiters which will be able to serve the arriving customers within satisfactory level of waiting time. The dominating and competing restaurants in Bangladesh are the ones with multi cuisines. In this regard, a case study of one such restaurant is presented at the pick hour of that restaurant.

The remaining portion of the work has been arranged and discussed as follows: literature review in section 2, a review of the core concepts of queuing theory used in this paper is put forth in section 3, section 4 presents detailed description and analysis of the case study, and finally, section 5 concludes the case study with the future scopes of this work.

2. Literature Review

Restaurant industry is emerging with a rapid growth rate and the researchers are viewing this field as a potential and important sector to carry out researches. Consequently, the literature extant on the restaurant industry is coterminous with its magnitude of importance. To enhance and ensure a superior level of customer satisfaction and experience, Roy et al. (2016) have proposed a queueing network model comprised of two levels and a nested semi open structure [1]. In another study, aesthetic view, design of a restaurant etc. are seen as significant factors in attracting customers and an assessment of these intangible features are made applying decision making trial and evaluation laboratory (DEMATEL) technique [8] . Research conducted in [5] explicated necessary relationships between various employee attributes and customer attributes by application of service profit chain model. This study reveals how employees treated in certain ways can have considerable amount of influence in customer satisfaction levels. Haghighi et al. (2012) considered customer loyalty to play a major role behind the success of a restaurant and therefore, evaluated several factors that have notable impact on the customer loyalty [9]. Data for this study was obtained through a set of questionnaires and was analyzed by implementing structured equation modelling.

The work presented in this paper encompasses the application of queuing theory to develop a queuing network which minimizes the total cost of operation along with the optimum number of servers at three different levels (counters, ovens and waiters) at a satisfactory level of waiting time. There are some instances of research works related to the application of the principles of queuing theory in restaurant design models. However, current literature provides no such works where cost is minimized alongside finding of optimum number of servers taking into account the waiting time. Jain & Ali (2016) used queuing theory to propose a model for the take away restaurants where simulation was performed and different types of statistical data (such as gain, loss etc.) was obtained [10]. Further studies by Patel et al. (2012) involved identification of parameters such as customer arrival rate, service rate, utilization rate etc. of a certain restaurant using data from the restaurant itself and applying Little's theorem and queueing model [6]. As restaurant business has become very popular and competitive in Bangladesh recently, studies in the context of this country are highly recommended. However, apart from the work by Ahsan et al. (2014), where an existing restaurant model was analyzed with the help of arena software and a proposed model was analyzed later using the same software that resulted in a reduced waiting time, the extant literature hardly involves any further relevant work in the context of this country [7]. In these circumstances, this case study is quite germane and could be an exemplary model for future works.

3. Review of the Basic Concepts

A queuing system is basically designed for the arriving customers to be attended by the limited number of servers. A queue forms when a customer arrives and finds the server busy. Moreover, this leads to the formation of different types of queuing systems that include single-server single-queue, multi-server single queue and a collection of single queues. These simple queuing systems have been illustrated in Fig. 2.

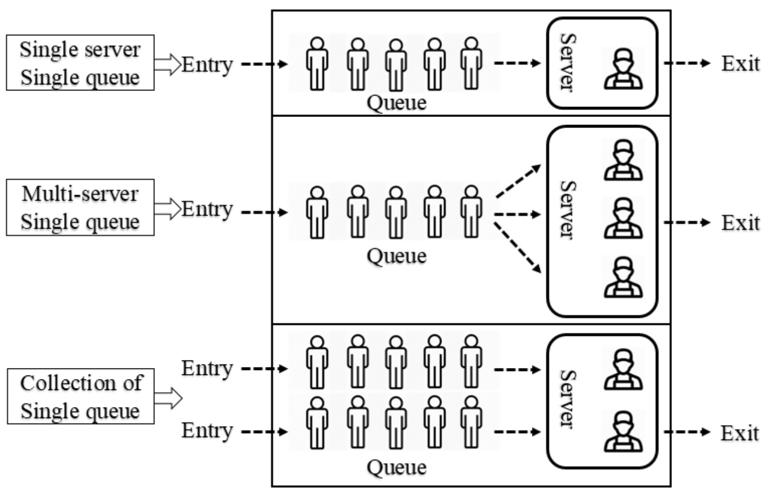

Fig. 1: Simple queuing systems

These queuing systems can be expressed as *M/M/s* using Kendall's notation, where the first *M* indicates the distribution of arrival process (Poisson distribution), the second *M* indicates the distribution of service time (Exponential distribution) and the last *s* indicates the number of parallel servers [11]; such notation is vastly used to solve queuing problems [12]. Relationship among arrival rate, service rate, cycle time and service time can be derived using the following Little's formula [13]:

$$L = \lambda T \tag{1}$$

where, L is the length of the queue, λ is the average customer arrival rate and T is the average service time which remains same regardless of the number of the customer arrival. Therefore, total number of the customers is proportional to the average rate of customer arrival. Using the Little's formula, necessary equations [14] for *M/M/s* model can be obtained as shown in Eqs. (2), (3), (4) and (5) which are used to determine the waiting time and queue length for optimizing the total cost in this study.

$$P_n = \frac{\left(\lambda/\mu\right)^n}{s!\,s^{n-s}} P_0 \tag{2}$$

$$L = L_q + \frac{\lambda}{\mu} = \frac{P_0 (\lambda/\mu)^s \rho}{s!\,(1-\rho)^2} + \frac{\lambda}{\mu} \tag{3}$$

$$W = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu} \tag{4}$$

$$q = \frac{\lambda}{s\mu} \tag{5}$$

4. Case Study

A reputed restaurant in Dhaka city serves different sorts of food items. The restaurant authority somehow managed their business being less concerned about the service quality provided to the customers so far, as it was the first such restaurant established in that area. Now the management of the restaurant has felt that as their customers are growing, somehow providing the service has become far more difficult than it was earlier. Moreover, dissatisfaction is arising among customers gradually because of their service quality. In such situation, their service facility should be upgraded as soon as possible to sustain in the race of burgeoning competition. Although a whole new up gradation requires huge amount of resources, management is firmly determined to do it; otherwise, they may end up losing their business markedly due to customer dissatisfaction. After studying the current system, management discerns that providing good service is considerably dependent on minimizing the waiting time starting from ordering to serving the prepared food which is sufficient enough for ameliorating the present scenario. This waiting time minimization is dependent on the modification of whole queuing network. This network is comprised of three queuing systems each of which corresponds to a service facility. Among these three queuing systems, first one is the queue of placing orders and paying bills; second one is for delivering the order to the corresponding oven while the third one is the prepared food waiting for a waiter available to be served it to the customer. For minimizing the waiting time in the first queue, certain number of servers is necessary to collect orders, receive payments and send the orders to the certain queues toward the specific oven according to the ordered items. It should be noted here that the restaurant wants to provide four new types of items for the customers including the existing nine. Therefore, for the second service facility in the network, certain number of ovens have to be established for each of thirteen different food items including the four newly added items. For the final service facility, certain number of waiters is to be employed so that the prepared foods are able to be served as fast as possible to their customers. Now, as there is a trade-off between losing business and constitution of the aforementioned service facilities in this network, the management team wants to do an Operations Research study to optimize the cost required while accounting for the customer satisfaction level. Required data to conduct the aforementioned OR study has collected through extensive study has been shown in table A and B in Appendix.

4.1 Description of Restaurant Service System Model

Any dine-in customer has to go through a nine-step process in the studied restaurant model. When a customer arrives at the restaurant, they are expected to wait in a queue in front of the billing counter to place the order and complete the payment. After placing the order, this information is passed to the kitchen immediately through the cloud-based server where the orders are categorized and allocated to the corresponding available oven. Afterwards, the chefs perform all the necessary activities to prepare the ordered items. Prepared items are then held in a queue to be picked by the waiters. Waiters deliver the items waiting in the queue to the respective customers which indicates the initiation of dining time. This whole process concludes when the customer leaves the restaurant. A flow chart of the entire dine-in process and a detail illustration of the proposed queuing network are shown in Fig. 2 and Fig. 3, respectively.

The dine-in process as shown from Fig. 2 clearly creates three queuing systems which eventually creates the queuing network as shown in Fig. 3. As seen from the Fig. 3, the queue created at the first step (Fig. 2) which is at the billing counter is denoted as Node-1 (service facility 1). The second queue is created at step 4 of Fig. 2 when the ordered items are categorized and queued to the dedicated ovens for that particular type of foods and this is marked as Node 2 in Fig. 3. Finally, at step 7 (Figure2) the prepared foods are queued to be picked by the waiters which is marked as node 3 in Fig. 3. Node 1 creates *M/M/s* queuing system as the customers wait in a single queue and multiple servers are assigned at the billing counter. However, at node 2 (service facility 2), we observe a number of parallel queuing systems, each consisting of single queue - multiple servers; thus, each parallel system follows an *M/M/s* system. Again, at node 3 (service facility 3), the prepared items wait in a single queue to be picked by any available waiter which can be expressed as another *M/M/s* model.

The restaurant layout includes kitchen space, dine in space, and washroom space. Dine in space includes tables with different seating capacities, aisles and cross aisles. So, the distance from the billing counter to table, service counter to table, table to exit may vary.

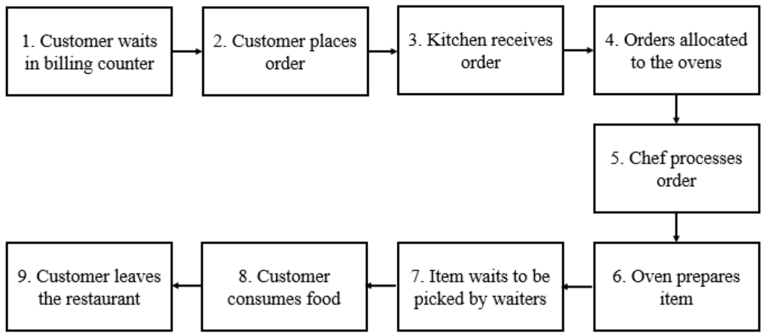

Fig. 2: Flow chart of the dine-in process

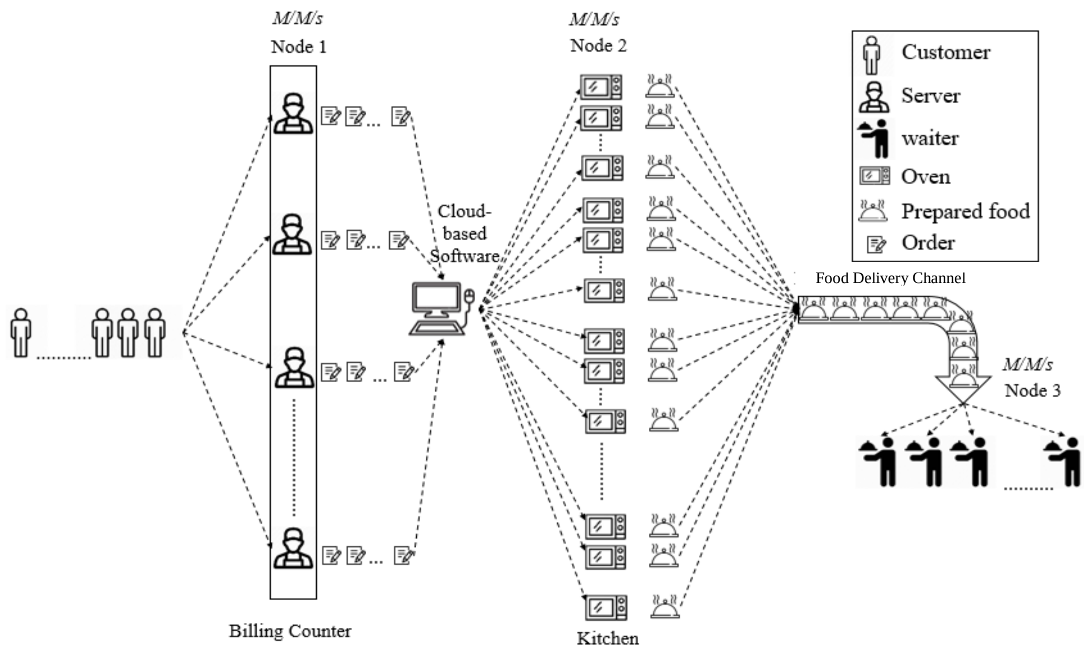

Fig. 3: Illustration of the queuing network of the restaurant model

4.2 Model Formulation and Solution

Assumptions:

Number of assumptions have been made to avoid complexity in the restaurant model which are:
1. Inter-arrival time has been assumed to follow Poisson distribution.
2. Service time distribution follows the exponential distribution.
3. Calling population has been assumed to be infinite.
4. Queue discipline follows first-come first-served (FCFS) method.

5. Queue length has been assumed to be infinite.
6. Mean demand and mean preparation time for a particular food item have been considered for calculation.
7. No delay between the 2nd node arrival time and the 1st node service time.

To minimize the expected total cost considering the queue at the nodes, the following objective function is formulated.

$$\mathrm{E(TC)} = \mathrm{E}(CS_1) + \mathrm{E}(Cq_1) + \mathrm{E}(CS_2) + \mathrm{E}(Cq_2) + \mathrm{E}(CS_3) + \mathrm{E}(Cq_3) \tag{6}$$

Here, the total cost is the summation of the cost incurred at billing counter, kitchen and the service counter. Eq. (6) can be expressed as follows:

$$(C_1S_1 + C_wL_1) + (\sum_{i=1}^{i=13} C_{2i}S_{2i} + C_w \sum_{i=3}^{i=13} L_{2i}F_{2i}) + (C_3S_3 + C_wL_3) \tag{7}$$

Here, the cost at billing counter includes the price of the digital machine and the salary of the corresponding operators as well as the cost of waiting expressed through queue length. The expense at the kitchen includes the cost incurred for the food items under the corresponding classes (represented by i) of foods considering both the operating cost and the waiting line. Finally, the cost incurred at the food delivering channel includes the salary of the waiters and again, the compensation for waiting in queue.

Besides objective function in Eq. (7), the optimization problem of the considered case contains the following constraints:

$$(W_1 + W_{2i} + W_3) \leq \textit{Corresponding Satisfactory waiting time}; \quad \forall\, i = 1, 2, 3, \dots \dots \dots \dots, 13 \tag{8}$$
$$q_1 < 1 \tag{9}$$
$$q_{2i} < 1, \qquad \forall\, i = 1, 2, 3, \dots \dots \dots \dots, 13 \tag{10}$$
$$q_3 < 1 \tag{11}$$
$$1 \leq S_1 \leq ub_1 \tag{12}$$
$$1 \leq S_{2i} \leq ub_{2i}; \quad \forall\, i = 1, 2, 3, \dots \dots \dots \dots, 13 \tag{13}$$
$$1 \leq S_3 \leq ub_3 \tag{14}$$

If we look at the constraints from Eqs. (8) to (14), we notice that there are actually three types of constraints: waiting time constraint (Eq. (8)), utilization factor constraint (Eqs. (9), (10) and (11)), and constraints limiting the server number (Eq. (12), (13) and (14)). Considering the first type of constraint, the total waiting time that includes the waiting time in queue for ordering the food, food preparation time and time taken by the waiters to serve, must be equal or less than the corresponding satisfactory waiting time to customers. Such satisfactory waiting time varies from food class to food class and values of them are chosen by the considered restaurant which they determine based on their business strategies. Note that in their orders customers can order foods from various food classes of which preparation time are different. However, in the constraint of Eq. (8), for W_{2i}, food class that consumes most time should be considered. Remember that we categorize the food classes, where each food class itself contains many food items, based on the ovens used to prepare them as well as their preparation time. In other words, if some foods can be prepared by same type of oven but have different approximated mean preparation time, those foods having different preparation time will be classified into different food classes. This perception can be discerned from the first column of Table 1 where though S_{21} and S_{22} both use same type of oven for being tandoori types of foods, still, are classified as different food classes due to their different preparation time.

In the above constraints, waiting time (W_1, W_{2i}, W_3) in various service facilities and their utilization factors (q_1, q_{2i}, q_3) can be determined with the help of Eqs. (2) to (5). Waiting time W_1, W_{2i}, W_3 can be determined using Eq. (4) which in turn require values of L_1, L_{2i}, L_3 from Eq. (3) while utilization factors q_1, q_{2i}, q_3 can be determined from Eq. (5) which in turn require values of $\lambda_1, \lambda_{2i}, \lambda_3$, μ_1, μ_{2i}, μ_3, S_1, S_{2i}, S_3 where S_1, S_{2i}, S_3 are decision variables. However, for calculating q_{2i}, one also need to consider the values of F_{2i}. As the considered decision variables need to be integers and the constraints are non-linear, the optimization problem should be solved by a non-linear integer programming approach. We have solved the optimization problem in MATLAB using genetic algorithm and the obtained result has been shown in the Table 1. The result shows the optimum number of servers and waiters needed which are 3 and 9, respectively. Note that number of waiters determined through our formulation represents number of waiters required in peak hours which may seem as idle resources for the demand other than the peak hours. This issue can be overcome by introducing proper scheduling techniques with the duty time of the waiters which is out of the scope of this paper. The obtained result also shows the number of ovens needed for each item which goes along with the considered demand at the peak hours. From the obtained results, it can be inferred that the items ordered frequently need more attention, and therefore they require more server than the other stations. The cost incurred in the objective function has been minimized to 953.7618 BDT per hour through this set up.

Table 1: Result obtained from the optimization model

Different types of servers in different service and sub-service facilities	**Numbers**
Order taking and conveying servers $- S_1$	3
Swarma mixed, chicken, beef, mutton tandoori specials $- S_{21}$	3
Fish tandoori specials $- S_{22}$	2
Tandoori ruti and porata $- S_{23}$	2
Beef gravy, chicken gravy, mutton gravy, fish gravy, sobjika khajana & daal $- S_{24}$	3
Pulao & biryani $- S_{25}$	2
Original Italian oven fresh pizza $- S_{26}$	2
Macaroni & tortellini $- S_{27}$	2
Salad sandwich and pizza sandwich $- S_{28}$	2
Original Italian pasta and pasta penny $- S_{29}$	1
Swarma wraps and pizza roll & garlic bread $- S_{210}$	2
Swarma & burger $- S_{211}$	3
French fries $- S_{212}$	2
Fresh juice, milk shake and coffee $- S_{212}$	1
Number of waiters $- S_3$	9

5. Conclusion

In this paper we present a comprehensive analysis of a real case of a restaurant from Bangladesh that depends on the queuing system design for providing better service to its customers. However, as the whole system of the restaurant practically contains three queuing system, it will be totally practical considering all of them as a single queue. Thus, to design the system we develop a queuing network consisting of three queuing systems with every detail that upholds the real scenario to be appeared in the considered restaurant according to its business strategy.

Although the analysis presented in this paper is with the real data from the considered restaurant, we cannot expose them as well as the identity of the restaurant for confidentiality concern in this competitive epoch. Also, detail calculation of evaluating the waiting time and utilization factor, especially for the service facility 2 is not shown here for space limitations. To mention some of the deviations from the practical scenario with a view to simplifying the proposed analytical model, note that the number of food orders under a certain food class may vary from day to day and hour to hour of the same day; however, the problem has been solved using the mean demand of all days for pick hours. However, idleness of the resources for other than the pick hours can be solved with proper scheduling techniques that entail one of our future works. In conclusion, it can be said that although setting up the required number of optimal servers at various service facilities requires the considered restaurant spending a good amount of money initially, the additional costs incurred for this new establishment are expected to be returned through a noticeable increase in sales by keeping the existing customers as well as attracting new customers within a short period as anticipated by in depth market analysis of the considered restaurant.

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